**Problem 1**

**A teacher samples 5 students from a class. Their test scores are: [72, 85, 78, 80, 69]. The teacher assumes the class mean score is around 75 with SD 10, but it's uncertain. Use Bayesian updating to infer the class average score.**

from scipy.stats import norm

import numpy as np

import matplotlib.pyplot as plt

# Prior: Normal(75, 10)

mu\_prior, sigma\_prior = 75, 10

# Sample data

scores = np.array([72, 85, 78, 80, 69])

sample\_mean = np.mean(scores)

sample\_std = np.std(scores, ddof=1)

n = len(scores)

se = sample\_std / np.sqrt(n)

# Update using conjugate prior (normal-normal)

precision\_prior = 1 / sigma\_prior\*\*2

precision\_sample = 1 / se\*\*2

posterior\_mean = (mu\_prior \* precision\_prior + sample\_mean \* precision\_sample) / (precision\_prior + precision\_sample)

posterior\_std = np.sqrt(1 / (precision\_prior + precision\_sample))

# Plot

x = np.linspace(60, 90, 1000)

posterior = norm.pdf(x, posterior\_mean, posterior\_std)

plt.plot(x, posterior, label=f'Posterior N({posterior\_mean:.1f}, {posterior\_std:.1f})')

plt.axvline(sample\_mean, color='red', linestyle='--', label='Sample Mean')

plt.axvline(mu\_prior, color='green', linestyle='--', label='Prior Mean')

plt.title("Posterior Estimate of Class Average Score")

plt.xlabel("Mean Score")

plt.ylabel("Density")

plt.legend()

plt.grid(True)

plt.show()

**Problem 2**

**You observe that 8 out of 10 randomly chosen students completed their homework. Based on experience, you think completion rate is usually ~70%. Estimate the posterior distribution of the true completion rate.**

from scipy.stats import beta

import matplotlib.pyplot as plt

import numpy as np

# Prior: Beta(7, 3) ~ belief centered at 0.7

a\_prior, b\_prior = 7, 3

# Data: 8 completions out of 10

completions = 8

total = 10

# Posterior

a\_post = a\_prior + completions

b\_post = b\_prior + (total - completions)

# Plot

x = np.linspace(0, 1, 1000)

posterior = beta.pdf(x, a\_post, b\_post)

plt.plot(x, posterior, label=f'Posterior Beta({a\_post}, {b\_post})')

plt.axvline(completions / total, color='red', linestyle='--', label='Observed Rate')

plt.title("Posterior of Homework Completion Rate")

plt.xlabel("Completion Rate")

plt.ylabel("Density")

plt.legend()

plt.grid(True)

plt.show()

**Problem 3**

**A student has taken 3 of 5 assessments and has an average score of 82 with a standard deviation of 4. Based on past performance, you believe students like them usually average around 78 with SD 6. Predict the final average grade using Bayesian inference.**

from scipy.stats import norm

# Prior: Normal(78, 6)

mu\_prior, sigma\_prior = 78, 6

# Observed: 3 scores, mean = 82, std = 4

mu\_sample = 82

n = 3

se = 4 / np.sqrt(n)

# Bayesian update (normal-normal conjugate)

precision\_prior = 1 / sigma\_prior\*\*2

precision\_sample = 1 / se\*\*2

posterior\_mean = (mu\_prior \* precision\_prior + mu\_sample \* precision\_sample) / (precision\_prior + precision\_sample)

posterior\_std = np.sqrt(1 / (precision\_prior + precision\_sample))

# Plot

x = np.linspace(70, 90, 1000)

posterior = norm.pdf(x, posterior\_mean, posterior\_std)

plt.plot(x, posterior, label=f'Posterior N({posterior\_mean:.1f}, {posterior\_std:.1f})')

plt.axvline(mu\_sample, color='red', linestyle='--', label='Sample Mean')

plt.axvline(mu\_prior, color='green', linestyle='--', label='Prior Mean')

plt.title("Predicted Final Grade Distribution")

plt.xlabel("Final Average Score")

plt.ylabel("Density")

plt.legend()

plt.grid(True)

plt.show()